1, 2, A Few, and Many

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Introduction

"1, 2, a few, many" are the only words some Australian Aborigines had for number. I used to think that quaint and couldn't image how they could live without words for all the numbers. I now look at that set of words differently for they fit the science of complexity even better than words for all the numbers.

The purpose of this article is to give a brief history of the development of complexity science, describe the different types of complexity, discuss examples of the types of complexity, and introduce some ideas about how complexity could be introduced into education.

History of Complexity Science

Numbers based science reached a milestone with Newton's laws for the behavior of 1 and 2 objects. He introduced the laws of motion and the law of gravitation in 1687. It wasn't until 1887 that Poincare, in his research on the three-body problem, became the first person to discover a chaotic deterministic system which laid the foundations of modern chaos theory. The three body problem was finally solved by Sundman in 1912 and was generalized to the case of $n > 3$ bodies by Qiudong Wang in the 1990s.

In the meantime number based science had jumped from 1 and 2 to many, where many is of the order of 10^{23} , bodies with the classical gas laws. It turned out that gases could be treated statistically, or as a whole, and gas laws similar to Newton's laws of motion could be developed. Boyle's law was developed in 1622, Charles law in 1787, Guy-Lussac's law in 1809, and Avogadro's law in 1811. Statistical methods were applied to ideal gases and that led to the development of the field of statistics.

Warren Weaver, in his 1948 landmark paper that introduced the concept of complexity, wrote in a discussion of problems of simplicity, "Speaking roughly, it mat be said that the seventeenth, eighteenth and nineteenth centuries formed the period in which physical science learned variables, which brought us the telephone and the radio, the automobile and the airplane, the phonograph and the moving pictures, the turbine and the Diesel engine, and the modern hydroelectric plant." He goes on to say that "the concurrent progress in biology and medicine was also impressive." But, because scientists in these fields couldn't limit themselves to one of two variables in isolation, they had to deal with the whole. "To sum up, physical science before 1900 was largely concerned with two-variable problems of simplicity; whereas the life sciences, in which these problems of simplicity are not so often significant, and had not yet become highly quantitative or analytical in character."

Weaver continued, "Subsequent to 1900 and actually earlier, if one includes heroic pioneers such as Josiah Willard Gibbs, the physical sciences developed an attack on nature of an essentially and dramatically new kind. Rather than study problems which involved two variables or at most three or four, some imaginative minds went to the other extreme, and said: 'Let us develop analytical methods

which can deal with two billion variables.' That is to say, the physical scientists, with the mathematicians often in the vanguard, developed powerful techniques of probability theory and of statistical mechanics to deal with what may be called problems of disorganized complexity."

Science basically skipped over the complicated problems of a few going from 1 and 2 to the many. Weaver wrote, "The new method of dealing with disorganized complexity, so powerful an advance over the earlier two-variable methods, leaves a great field untouched." But while number is a convenient way to think about the three regions of science, it is not the only way. "The really important characteristic of the problems of this middle region, which science has as yet little explored or conquered, lies in the fact that these problems, as contrasted with the disorganized situations with which statistics can cope, show the essential feature of organization. In fact, one can refer to this group of problems as those of organized complexity" wrote Weaver.

Many writers have characterized this new area of science as critical to our future, starting with Weaver:

- "These new problems, and the future of the world depends on many of them, requires science to make a third great advance, an advance that must be even greater than the nineteenth century conquest of problems of simplicity or the twentieth century victory over problems of disorganized complexity. Science must, over the next 50 years, learn to deal with these problems of organized complexity."

- Sanders and McCabe quote Hawking in their report, "I think the next century will be the century of complexity", Stephen Hawking, January 2000. They go on to write, "The challenges of the 21st century will require new ways of thinking about and understanding the complex, interconnected and rapidly changing world in which we live and work. And the new field of complexity science is providing the insights we need to push our thinking in new directions.

    In the last twenty years, rapid advances in high-speed computing and computer graphics have created a revolution in the scientific understanding of complex systems. We now have the ability to move beyond the old reductionist paradigm; to look at whole systems; to study the interactions of many interdependent variables and to explore the underlying principles, structure and dynamics of complex physical, biological and social systems.

    From health care to city planning to economics and international politics, the new science of complex systems is moving us away from a linear, mechanistic view of the world to one based on nonlinear dynamics, evolutionary development and systems thinking. It is laying the foundation for a fundamental shift in how we view the world, and with it the need for a shift in how we think about, organize, plan for, and lead 21st century organizations."

- In "Complexipacity", David Snyder quotes Stafford Beer as saying, "the over arching challenge of our age will be "managing modern complexity".

- And Heinz Pagels, physicist, is quoted by Waldrop as saying, "I am convinced that the nations and people who master the new sciences of complexity will become the economic, cultural, and political superpowers of the next century."

- Zimmerman, Lindberg and Plesk write about a different but very import aspect of complexity science, “Complexity has created a bridge or a merger of quantitative and qualitative explanations of life. It has attracted some of the greatest thinkers in the world including some of the most highly respected organization theorists and Nobel Prize winners in physics, mathematics and economics. It has also attracted poets, artists and theologians who see the optimism implicit in the science. By examining how life happens from a complexity perspective, we seem to have increased our reverence for life - the more we understand, the more we are amazed.”

It is difficult to write only a few paragraphs about the development of the science of complexity. Most writers credit Lorenz. Greg Rae has written a very good short introduction to chaos theory in his blog (IMHO In My Humble Opinion). He writes, “The first true experimenter in chaos was a meteorologist, named Edward Lorenz. In 1960, he was working on the problem of weather prediction. He had a computer set up, with a set of twelve equations to model the weather. It didn't predict the weather itself. However this computer program did theoretically predict what the weather might be.

One day in 1961, he wanted to see a particular sequence again. To save time, he started in the middle of the sequence, instead of the beginning. He entered the number off his printout and left to let it run.

When he came back an hour later, the sequence had evolved differently. Instead of the same pattern as before, it diverged from the pattern, ending up wildly different from the original. Eventually he figured out what happened. The computer stored the numbers to six decimal places in its memory. To save paper, he only had it print out three decimal places. In the original sequence, the number was .506127, and he had only typed the first three digits, .506.

By all conventional ideas of the time, it should have worked. He should have gotten a sequence very close to the original sequence. A scientist considers himself lucky if he can get measurements with accuracy to three decimal places. Surely the fourth and fifth, impossible to measure using reasonable methods, can't have a huge effect on the outcome of the experiment. Lorenz proved this idea wrong.”

This type of effect became known as the “Butterfly Effect”. Gleick writes, “The Butterfly Effect acquired a technical name: sensitive dependence on initial conditions. And sensitive dependence on initial conditions was not an altogether new notion. It had a place in folklore:

“For want of a nail, the shoe was lost;
For want of a shoe, the horse was lost;
For want of a horse, the rider was lost;
For want of a rider, the battle was lost;
For want of a battle, the kingdom was lost”

An important development in the history of the science of complexity was the creation of the Santa Fe Institute. M. Mitchell Waldrop devotes an entire book, ***Complexity: The Emerging Science at the Edge of Order and Chaos*** to the creation and development of the Santa Fe Institute. Melanie Mitchell traces the history of the development of complexity in computer science, real and artificial life in her book, ***Complexity: A Guided Tour***. And James Gleick in ***Chaos: Making a New Science*** devotes an entire book to the development of the science of chaos.

In truth, the science came from innumerable sources:

- Mathematicians studying nonlinear equations
- Physicists attempting to solve the three body problem
- Medical researchers discovering that the normal heart rhythm is chaotic
- Biologists discovering the African termite cathedrals are built by a nest of termites each following very simple rules
- Meteorologists' attempting to solve the weather prediction problem
- Geologists discovering that earthquake strength and frequency of occurrence follows a simple inverse square law
- Geometers discovering fractals in nature
- Everyone looking at flocking birds and schooling fish and wondering how and why.
- Evolutionary biologists exploring evolution
- Chemists discovering chemical clocks
- Physicists studying Raleigh-Bernard convection
- Biologists studying the slime mold
- Ecologists studying systems in nature
- Theoretical physicists modeling the hourglass
- Economists studying real world economies
- Game theorists
- Studies of how cities develop
- Poets, writers, philosophers and artists have expressed it

This is just a sampling, by no means a complete list. The science of complexity touches the real world we interact with that has remained hidden from our perception until now. It not only touches every discipline but alters perception of each. And, complexity science transcends the disciplines. In that sense it is manifesting emergence, one of effects enumerated in the science. The science of complexity is emerging from the disciplines with a power greater than the sum of the disciplines. It is trans-disciplinary.

Types of Complexity

Even though the science of complexity has been emerging for over 50 years, there isn't a definition of complexity that is universally accepted. Melanie Mitchell in ***Complexity*** writes, "…it has been surprisingly difficult to come up with a universally accepted definition of complexity". In 2004, she organized a panel discussion on complexity at the Santa Fe Institute's annual Complex Systems Summer School. Her first question to a panel of faculty members of the Institute was "How do you define complexity?" Each panel member gave a different answer to the question. As frustrating as this is, the reason is that there are many sciences of complexity. The work is still to be done to reach a general science of complexity.

Mitchell writes, "The physicist Seth Lloyd published a paper in 2001 proposing three different dimensions along which to measure the complexity of an object or process:

1. How hard is it to describe?
2. How hard is it to create?
3. What is the degree of organization?"

She then lists the following possible definitions of complexity:

- Complexity as size
- Complexity as entropy
- Complexity as algorithmic content
- Complexity as logical depth
- Complexity as thermodynamic depth
- Complexity as computational capacity
- Statistical complexity
- Complexity as fractal dimension
- Complexity as degree of hierarchy

Each of these, she goes on to demonstrate, are valid for certain types of complexity but not generally applicable.

As long as this list is, I don't think it's the complete list of suggested definitions.

This is not at all surprising. It's the normal way for science to develop: special theories transforming into general theories.

For the purposes of this discussion, which is not to advance the science of complexity, but to tease out the implications for education, I'm going to make some simplifying assumptions and then look at the characteristics of the types of complexity that remain.

First, I'm going to accept Weaver's classification of simplicity, organized complexity and disorganized complexity. We educate people well in simplicity and do a fair job with disorganized complexity. The topic for the rest of this discussion will be focused on organized complexity.

Within the topic of organized complexity, I propose three categories – complicated, chaos and criticality.

Critical Systems

A complex system in a critical state is a system with a large number of members interconnected with others through historical events. It is a system in non-equilibrium.

One of first complex critical systems studied was earthquakes. A fault line is not a line but a system of faults, a cascading network of faults. The system of faults has been cause by the slippage of one tectonic plate against another tectonic plate. Earthquakes have resisted any attempts of forecasting them – time or magnitude. Mark Buchanan, ***Ubiquity***, relates the story of how two disciplines merged to provide an answer:

"...Yakov Kagan was giving a more or less routine lecture on earthquakes, and, as most scientists attending were not geophysicists, he was offering a general overview. Kagan related the sad tale of the singular failure that he and his colleagues continued to meet in trying to forecast earthquakes. And he also introduced his audience to one of the few hard and fast laws ever discovered about earthquakes, a rule describing how often earthquakes of various magnitudes take place. The rule is known as the Gutenberg-Richter law. The law reveals that larger earthquakes take place less frequently than smaller, and what's more, that the precise relationship is known to mathematicians as a power law – a special mathematical pattern which has a simplicity that stands in shocking contrast to the overall complexity of the earthquake process."

If the energy of the earthquake doubles, it becomes four times less likely to occur. This law holds over a very wide range of earthquakes:

$F = C\,M^{-2}$, where F is the frequency of occurrence, C is a constant and M is the magnitude of the earthquake.

As it turned out, the insight had been demonstrated since the middle of the third century in the hourglass.

Per Bak, Chao Tang, and Kurt Weissenfeld, were working on understanding how the sand pile grew in the hourglass. Sometimes a grain of sand will fall and role down the pile. At other times it will stick. And other times it will cause an avalanche of sand grains. They had built a computer model of the sand pile and had found similar phenomena. What they discovered was that a critical state occurred easily in this simple model. The growing pile was always in non-equilibrium. As a result a single grain could cause an avalanche of hundreds or thousands of grains. In short they had discovered another power law.

Bak was in the audience listening to Kagan and excitedly wondered if they could build a model of earthquakes that would show the power law effect. They were able to do this by simplifying

the assumptions carefully for a computer simulation.

Many other complex critical systems have been discovered such as forest fires, grasshopper plagues, measles epidemics, operation of pulsars, mass extinctions, etc.

A complex system in a critical state has some surprising characteristics:

- Cause and effect are disconnected. As Buchanan wrote, "...that the greatest of events have no special or exceptional causes."
- A power law can only be generated by some process that is steeped in history. "...the future emerges out of a string of accidents, each leaving its indelible trace on the course of events," Buchanan commented.
- The complex system in a critical state exhibits the property of self similarity or scale invariance. It looks the same at all scales. Again, referring to Buchanan, "...the Gutenberg-Richter power law says that the process behind earthquakes is scale invariant, and the unavoidable implication is that the great quakes are no more special or unusual than the tiny shudders constantly rippling beneath our feet."
- Systems that exhibit self organizing critically are ubiquitous
- A new type of statistics has to be applied to complex systems in a critical state. They do not follow normal, i.e. Gaussian, statistics. Large events are a lot more likely to occur than a normal distribution would predict. Nassim Nicholas Taleb wrote in ***The Black Swan,*** "A Black Swan is a highly improbable event with three characteristics: It is unpredictable; it carries a massive impact; and after the fact, we concoct an explanation that makes it appear less random, and more predictable than it was."
- The results of attempts to "improve" the behavior of a complex system in a critical state are unpredictable and can actual make the performance worse. Since the 1890's the forest service had a policy of stamping out a forest fire as quickly as possible. The logic behind that policy was that it was better to catch a fire small before it became big. In 1998 the Yellowstone fire consumed almost 800,000 acres, 36% of the park, and the number and intensity of forest fires was increasing everyehere. Also in 1998, the geologists Bruce Malamud, Gleb Morein, and Donald Turcotte of Cornell University gathered extensive data on forest fires. They demonstrated that the number of acres consumed in a forest fire followed an inverse power law of 2.48. A forest is a complex system in a critical state, at least for forest fires. The actions of the forest service were inadvertently making the critical state more dangerous by not allowing the small naturally occurring fires to keep the system in its natural state.

And, that leads us to a brief overview of adaptive critical state systems. All the examples mentioned to this point had parts of the system that were not intelligent agents. So, do complex systems in a critical state made up of intelligent agents behave the same way? All the indications to date support a yes.

We humans seem to have a propensity to organize ourselves into critical states. There is evidence that the stock market's variability obeys a power law and that variability exhibits scale invariance. "This flies in the face of the efficient market hypothesis "reports Buchanan, "and that the markets are in equilibrium."

I looked at the day to day changes in stock market in 2007 and 2008. As you remember, 2008 was a bad year for the stock market. The question I was curious about was whether 2008 a different kind of behavior than 2007? The answer was no. Using the S&P 500 day to day variation, both 2007 and 2008 had an inverse power law of 2.1. The same complex system in a critical state was at work in both cases. I also checked it with 2000 and 1994 with no change in my conclusion.

I was also curious about technological innovation. I choose the integrated circuit for my example because we have such good data on the history of its development. What I was looking at was the magnitude of the change in circuits per chip between product announcements. To my surprise, this also followed a power law of 1.5. Not as high a coefficient as earthquakes or stock markets, but never the less, a power law.

Both the stock market and IC development examples have strong simple trends that might extend out over decades. But the fine detail is indicative of an operating critical state system.

The human based systems do not yet have all the research supporting the assumption that they have the same characteristics as non-adaptive agent critical state systems, but they appear to.

At the close of ***Ubiquity***, Buchanan wrote, “Like chaos, the critical state bridges the conceptual gap between the regular and the random. The pattern of change to which it leads through its rise of factions and wild fluctuations is neither truly random nor easily predicted. It is a universal and understandable pattern that none the less slips the grasp of detailed prediction, reveals only in the statistics and draws the human mind into conceptual error. It does not seem simple and law like for long periods of calm to be suddenly and sporadically shattered by cataclysm, and yet it is. This, it seems, the ubiquitous character of the world.”

Chaotic Systems

Chaos is often misunderstood with respect to complexity. Chaos has a meaning outside of complexity: confusion and disorder. But, in complexity is has a meaning closer to the one from 3000 years ago. Hesiod described a trinity of forces in his book, ***Theogony***, at work in the world. This trinity of forces has been carried down through the ages, appearing in various forms. Now more commonly known as the Orphic trinity, this tradition is traced through the ages in a great book by Ralph Abraham, ***Chaos Gaia Eros***, to modern science. Abraham writes, “The characteristic features of this tradition are the trinity:

- Chaos, the creative void, source of all form
- Gaia, the physical existence and living spirit of the created world
- Eros, the spiritual medium connecting Chaos and Gaia, the creative impulse.”

The science of complex chaotic systems is finding that all form, and possible life itself originates from the edge of chaos. Chaos can exist in systems like the double pendulum, apparently simple equations like the logistic map, the weather, and the beating heart. Complex chaotic systems can be adaptive or non-adaptive.

Non Adaptive Complex System

A non-adaptive chaotic system is a dynamical system (a process in motion) that is highly sensitive to initial conditions, often making the system to appear random. However, the chaotic system is deterministic, meaning that future states of the system are determined by the initial conditions. However, it is aperiodic (irregular or without periodicity).

The non adaptive example often used to demonstrate chaos is the logistic map. The logistic equation originated in population changes is deceptively simple. It usually takes the form:

$X_{n+1} = R X_n (1 - X_n)$, where R is a constant, X is value, and n is the iteration number.

This equation produces bifurcations at various levels of R, much like the numerical sequence, 1, 2, 4, 8, etc (powers of 2). When it reaches one of the bifurcations, the results oscillate between the various possible results, never reaching a limit. The equation converges to a single value until R = 3. At that point it begins to oscillate between two values. It bifurcates again at R = 3.44949 and oscillates between four values. At R = 3.569946, it oscillates between an infinite number of values.

This equation is very sensitive to initial conditions. Very small changes in X_0 = result in significantly different results, although the period doubling always occurs at the same values of R. Also, there some values of R between 2 and 3.569946 where relatively little is happening. A closer examination of those regions indicates that they contain a microcosm of the entire sequence.

Robert May, a biologist, studied the logistic map. James Gleick, ***Chaos***, reports May's findings this way:

“These bifurcations would come faster and faster – 4, 8, 16, 32... - and suddenly break off. Beyond a certain point, the 'point of accumulation', periodicity gives way to chaos, fluctuations that never settle down at all. Whole regions of the graph are completely blacked in. If you were following an animal population governed by the simplest of nonlinear equations, you would think the changes from year to year were absolutely random, as though blown about by environmental noise. Yet in the middle of this complexity, stable cycles suddenly return. Even though the parameter is rising, meaning that non-linearity is driving the system harder and harder, a window will appear with a regular period...

At first May could not see the whole picture. But the fragments he could calculate were unsettling enough. In a real world system, an observer would see the just vertical slice corresponding to one parameter at a time. He would see only one kind of behavior – possibly a steady state, possibly a seven year cycle, possibly apparent randomness. He would have no way of knowing that with the same system, with some slight change to some parameter, could display patterns of a completely different kind.”

The bifurcation diagram is a fractal: if you zoom in on the above mentioned value R = 3.82 and

focus on one arm of the three, the situation nearby looks like a shrunk and slightly distorted version of the whole diagram. The same is true for all other non-chaotic points. This is an example of the deep and ubiquitous connection between chaos and fractals.

If you roll a marble inside a bowl, it will oscillate around the bottom of the bowl until it comes to rest at the bottom. This is the attractor. In chaotic systems there may be many attractors but the system never comes to rest on any one of them. In this equation the attractors are the bifurcations.

Chaotic behavior has been observed in the laboratory in a variety of systems including electrical circuits, lasers, oscillating chemical reactions, fluid dynamics, and mechanical and magneto-mechanical devices. Observations of chaotic behavior in nature include the dynamics of satellites in the solar system, the time evolution of the magnetic field of celestial bodies, population growth in ecology, the dynamics of the action potentials in neurons, and molecular vibrations. Everyday examples of chaotic systems include weather and climate.

“Chaos is everywhere. Perfect systems may be easily modeled according to the 'laws of physics': with the mass less ropes, frictionless surfaces and perfect vacuum of physics text-book problems. Real systems have friction, air-resistance and physical variations that make them unpredictable.” Writes Lesa Moore, Macquarie University

A non-adaptive chaotic system can have the following properties:

- Unpredictability: Unpredictability is not randomness, but in some circumstances looks very much like it.
- Determinism: But it's a peculiar type of determinism. It is not the determinism of classical physics. Gleick explains, “The system is deterministic, but you can't say what it's going to do next.”
- Sensitivity to initial conditions: a small change in the initial conditions can lead to significant changes later. This effect limits the accuracy of weather forecasts
- Attractors: not all chaotic systems have attractors; some are chaotic everywhere
- Non-linearity
- Self similarity: similar to fractals. Some people even equate fractals as the geometry of chaos.

Melanie Mitchell summarizes the radical new learning from chaotic systems in the following way. “...some of these new ideas, which few nineteenth-century scientists would have believed.

- Seemingly random behavior can emerge from deterministic systems with no external source of randomness.
- The behavior of some simple, deterministic systems can be impossible, even in principle, to predict in the long term, due to sensitive dependence on initial conditions.
- Although the detailed behavior of a chaotic system cannot be predicted, there is some order in chaos seen in universal properties common to large sets of chaotic systems...Thus even though prediction becomes impossible at the detailed level, there are some higher level aspects of chaotic systems that are indeed predictable.”

Fractals

Fractal is a term coined by Benoit Mandelbrot in 1975 and was derived from the Latin fractus meaning "broken" or "fractured." A mathematical fractal is based on an equation that undergoes iteration, a form of feedback based on recursion. A fractal can be a simple geometric figure, or they can be complex. A fractal can geometrically describe the logistical equation discussed earlier.

Fractals usually have the following properties:

- Fine structure at arbitrarily small scales
- Too irregular to be easily described in traditional Euclidean geometric language.
- Self-similarity
- Fractional dimensions
- A simple and recursive definition

The Mandelbrot set is based on an equation similar to the logistic equation:

$X_{n+1} = X_n^2 + C$, where C is a complex number.

What emerges from the repeated application of this simple equation when graphed in color are images great depth and beauty. The depth depends upon how many iterations are carried out. Diving deep into one of the dendrites of the Mandelbrot set is like being a voyeur to creation at the scale of galaxies or atoms.

Adaptive Chaotic Systems

Adaptive chaotic systems pose new complications because the members of this type of systems learn, and quite often are alive. And, while some non-adaptive chaotic systems can produce emergence, emergence is usually associated with adaptive chaotic systems. Emergence is the way complex systems and patterns arise out of a multiplicity of relatively simple interactions.

We've observed these types of systems for years in the behaviors of social insects, flocking of birds, schooling of fish, growth of cities, behavior of slime mold, herding and migratory behaviors, among many others.

In 1979 I saw a lecture by Phillip Morrison, an MIT professor, on PBS entitled "Termites to Telescopes". The implications of that lecture have haunted me ever since. Morrison described how African termites build very large and complex mounds for their nests.

Communication among termites is not completely understood. Since they live and work in darkness, they are blind, as we know the term. Smell and touch seem to be the preferred form of communication. Termites build nests from a material that they make with body chemicals and cellulose, wood fiber. Big termite nests, like those found in Africa or Australia can be ten or twelve feet high and last decades. A nest may contain millions of individuals. Termites require carefully controlled humidity and temperature conditions inside the nest. The structure and

material provide this function. Function and form are in consonance.

Construction of a nest follows a simple procedure. At some point for reasons unknown, and by mechanisms unknown, upon sensing a "signal" of some sort, termite workers start producing the pellets of material they use to construct nests. The termites begin to pile these pellets, each working individually, cementing them together with an adhesive they produce.

At some later time, sensing another "signal," the workers "look" around them. If they see a pile of pellets larger than theirs in the immediate vicinity, they abandon their project and go work on the higher pile. Through this process they select those piles they will work on.

A little while later, sensing still another "signal," the workers "look" around to see if there is a pile of nearly the same height within a specified distance of the pile they are working upon. If not, they abandon their pile and search for two piles that are close together. Again, after time has elapsed, termite workers begin to form the arch at the top. This process is repeated many times until an interlocking web of randomly constructed arches is completed.

In this process there are no high-performing termites. The entire process can be written in the form of a set of simple logical instructions - a program. There is no plan and no leader. Randomness plays an important role. The instructions and the responses seem to be genetically programmed into the termite worker. Signals do not seem to be given by anyone. Environmental conditions dictate the start of the process. When it is time to build a nest, a nest is built. The processes can be defined logically, analytically. Time may even play a role in the behavior changes once the building has begun. No one has a vision of the outcome. Everyone follows the rules and the result is functionally correct, but maybe not elegant by human standards.

These types of systems lend themselves well to a new type of modeling not based on equations. Massively parallel modeling with intelligent agents can simulate these systems well. The simplest elements are a space with properties for the intelligent agents to operate on and simple rules for the agents to follow. Sometimes randomness can be added in as well as more complicated properties of the agents, and different types of agents. The purpose of this type of modeling is not to predict outcomes (the systems are unpredictable) but to provide insight and understanding. Two modeling systems are easily accessible – StarLogo and The Game of Life.

It's interesting to note that this type of system uses simple rules to create complexity in a state of equilibrium. Whereas, other systems, established by a set of simple rules, create complexity in a state of critically.

The characteristics of adaptive chaotic systems are:

- Generated from simple rules
- Massively parallel processes
- Intelligent agents
- Non deterministic
- Unpredictable
- Exhibit emergent properties

- Decentralized

Complex Adaptive Systems (CAS) is a large field in itself. In this field people are studying evolution, life, and artificial life. The emerging field is a synergistic combination of life sciences and computer science. It is best describe in Melanie Mitchell's book, ***Complexity a Guided Tour***. Mitchell describes four principles of adaptive information processing in decentralized systems:

1. Global information is encoded as statistics and dynamics of patterns over the system's components
2. Randomness and probabilities are essential
3. The system carries out a fine-grained, parallel search of possibilities
4. The system exhibits a continual interplay of bottom-up and top-down processes.

At the close of the book, Mitchell writes, “While much of the science I've described in this book is still in it's early stages, to me, the prospect of fulfilling such ambitious goals is part of what makes complex systems a truly exciting area to work in. One thing is clear: pursuing those goals will require, as great science always does, an adventurous intellectual spirit and a willingness to risk failure and reproach by going beyond main stream science into ill defined and uncharted territory. In the words of the writer and adventurer Andre Gide, ‘One doesn't discover new lands without consenting to losing sight of the shore’.”

Complicated Systems

A complicated system is one that is nothing but the sum of its parts, and that an account of it can be reduced to accounts of individual constituents. The nature of a complicated system can be found by reducing the system to the interactions of its parts, or to simpler or more fundamental things.

A complicated system does not have the properties of a chaotic or critical system. It usually does not have the properties of a disorganized complex system. A complicated system is organized. Theoretically, you can take a complicated system apart and then put it back together again and it will operate as before.

Examples can be found in a variety of areas. Some examples are:

- Human design: A car is a complicated system. It may have over 10,000 parts and 50 of those parts may be microprocessors. Probably no one person knows every part in a car and what its function is. That's because the function of the car is hierarchical. It is broken down into subsystems and each subsystem is designed independently to integrate with all the other subsystems to produce the desired functionality of a car. However, the user interface is simple. You need to know very little and interact with only a few elements to drive it. Jeffery Kluger, ***Simplexity***, complains about the opposite extreme, when the product and the human interface are complicated. “Electronic devices by any rational measure have gone mad. It's not just your TV or your camera or your twenty-seven-button cell phone with its twenty-one different screen menus and its 124 page instruction manual. It's your camcorder and your stereo and your BlackBerry and your

microwave and your dishwasher and your dryer and your new multifunction coffeemaker, which in any sane world would have just one job to do and that's to make a good cup of coffee." With the complications shifted to the buyer, the cost of design is reduced, and the devices are so complicated that many have to pay someone else, possibly even the company that designed it, to set it up and operate it, generating more revenue. Is this complication by design intentional?

- Law: I don't know how to characterize how law is developed. In a pure democracy it would be a chaotic system with intelligent agents working to allow the law to emerge. But, it's not a pure democracy. What ever the process, the result is a complicated system. The U.S. Federal Tax Code is so complicated that people have to go to school to learn it every year. This complication stacks the deck for people and businesses who can afford to pay or hire some one (or a staff) to find the advantages for their client. Christopher Beam, writing about one of the health care bills being discussed in the US Senate, reports, "It's now 1,018 pages, to be exact. Is that especially long for a bill? Not really. Sure, most legislation is much shorter: The average statute passed by the 109th Congress - the latest session for which figures are available - clocked in at around 15 pages, according to the Senate Library. And the recent law authorizing President Obama to give gold medals to the Apollo 11 astronauts on the 40th anniversary of the moon landing filled just two pages. But major spending bills frequently run more than 1,000. This year's stimulus bill was 1,100 pages. The climate bill that the House passed in June was 1,200 pages. Bill Clinton's 1993 health care plan was famously 1,342 pages long. Budget bills can run even longer: In 2007, President Bush's ran to 1,482 pages."
- Bazaars: A bazaar is merchandising area, marketplace, or street of shops where goods and services are exchanged or sold. The roots of the word imply a place of prices. Today's grocery stores are bazaars. A typical grocery store contains 45,000 items. Picking and chooseing what you want to purchase from all this variety is complicated. There are roughly 10^{47} ways to pick a dozen items to buy. Modern shoppers handle this complication with ease although the solution they find may not be optimum. Experience in the bazaar teaches people strategies to minimize time, cost or to get what they want. Merchandizing is the way stores and suppliers try to simplify the complicatedness, and have you buy what they want you to buy.
- Technology: Technology, hardware and software, provides some complicated examples. Windows 95 had 15 million lines of code. That grew to 18 million lines by the time Windows 98 launched. Windows XP, released in 2001, has 35 million lines of code. Windows Vista, more than five years in the making, has more than 50 million lines of code. These are complicated systems and the methods of developing and managing them is the source of competitive advantage. (By way of example, a typical book might contain 70,000 words – roughly 5,000 lines.) The number of transistors per integrated circuit chap has grown from about 2,000 in 1971 to almost 2 billion now. The systems of the design, manufacture and testing of billions of transistors are indeed complicated.

Introducing Complexity into Education

The challenge of complexity science is great. Not only does the science need to advance in pursuit of a unified theory of complexity, but we have to determine how we are going to educate our societies about complexity, educate them and develop practical applications of

complexity in all aspects of life.

Zimmerman, Lindberg and Plesk write, “Complexity has created a bridge or a merger of quantitative and qualitative explanations of life. It has attracted some of the greatest thinkers in the world including some of the most highly respected organizational theorists and Nobel prize winners in physics, mathematics and economics. It has also attracted poets, artists and theologians who see the optimism implicit in the science. By examining how life happens from a complexity perspective, we seem to have increased our reverence for life – the more we understand, the more we are amazed.”

There are at least three areas for complexity science in education: the application of complexity science to the way people learn, understanding of the education system as a complex system, and the teaching of complexity science.

The first two are outside the scope of this work. (See Complexity and Education for more information) The latter is also in its entirety outside the scope of an essay like this. However, I will introduce an approach including some suggestions on common concepts, special concepts, strategies and multidisciplinary focus areas:

Common Concepts

The common concepts across all complex systems, disorganized and organized (complicated, chaotic, critical), are:

- Systems [1]: We need to teach:
    - What a system is
    - The types of systems
    - How one determines the type of a system
    - When do you have enough of the system included
    - How does a system respond to its environment
- Reductionism and holism[2]: Both approaches need to be taught along with when each one should be applied.

[1] : “System” derives from a very rich Indo-European source, “sta”, which meant to stand or stay. Hundreds of English words are derived from this source. System came through the Greek “histanai”, to cause to stand. When combined with the Greek prefix “sun”, which meant with, the word became “sunistanai”, to place or set together. This became “sustema”, a number of things placed together. It was changed to “systema” in Latin, and finally “system” in English. There is a type of duality present. A system is a number of things that stand/stay together. “Sta” came through German to become “gestalt”. Gestalt is a German word for form or shape. It is used in English to refer to a concept of wholeness. The concept of wholeness has an important role in a system. Gödel, the German mathematician, proved that any closed system (of language, mathematics, thought, physics) always has residual error. The fact that we put a boundary around a system will always result in error. Our journey of knowledge is always to include more elements into the systems we consider.

[2] “Reductionism can either mean (a) an approach to understand the nature of complex things by reducing them to the interactions of their parts, or to simpler or more fundamental things or (b) a philosophical position that a complex system is nothing but the sum of its parts, and that an account of it can be reduced to accounts of individual constituents. A contrast to the reductionist approach is holism or emergentism. Holism recognizes the idea that things can have properties as a whole that are not explainable from the sum of their parts (emergent properties). The principle of holism was concisely summarized by Aristotle in the Metaphysics: 'The whole is more than the sum of its parts'.” Wikipedia.

- Statistics: Statistics is a branch of mathematics concerned with collecting and interpreting data. Depending on the system different types of statistics need to be used. People need to know how to collect and interpret data taken from all types of complex systems.
- Pattern recognition: Steven Johnson, ***Emergence***, writes, "As the futurist Ray Kurzweil writes, 'Humans are far more skilled at recognizing patterns than in thinking through logical combinations, so we rely on this aptitude for almost all of our mental processes. Indeed, pattern recognition comprises the bulk of our neural circuitry. These faculties make up for the extremely slow speed of human neurons.' ...the brain is a massively parallel system, with 100 billion neurons all working away at the same time." We need to educate people in how to use this capability, at a conscious level, to see patterns in systems or to use tools to assist in that recognition process.

Chaos and Critical Systems Concepts

There are several important concepts for chaotic and critical systems. The first three have already been examined earlier.

- Chaos
- Criticality
- Emergence
- Decentralization: "As we enter the Era of Decentralization, there is an important educational challenge: How can we help people become intellectually engaged with the new types of systems and new ways of thinking that characterize this new era? To date, schools and other educational institutions have done little, if anything, to engage students with the idea of decentralization. Instead, they often perpetuate centralized explanations and approaches" comments Mitchel Resnick, ***Turtles, Termites, and Traffic Jams***. He develops five heuristics for decentralized thinking:

    - Positive feedback isn't always negative
    - Randomness can help to create order
    - A flock isn't a big bird
    - A traffic jam isn't just a collection of cars
    - The hills are alive

  "There is an apparent paradox in people's reactions to decentralized systems." Resnick explains. "On one hand is the allure of decentralization. They are fascinated by systems that are organized without an organizer, coordinated without a coordinator." On the other hand, "When people see patterns in the world, they intuitively assume that the patterns are created by lead or by seed." That's the educational challenge.

  Sanders and McCabe write, "Complex adaptive systems, and models thereof, are characterized by distributed organizations or networks, whose parts all influence each other, either directly or through feedback loops, which continually evolve and adapt to accomplish overarching goals. This is in fundamental contrast to the top-down, hierarchical management structures found in most government organizations and in much of corporate America, where local experimentation, innovation and adaption are

discouraged in favor of rigid bureaucratic rules and planning procedures. Simple cause and effect relationships do not characterize complex adaptive systems, and hence most of the conventional policy-planning tools currently used by decision-makers, both government and corporate, are inappropriate and ineffective.

...Complexity science offers new ways of understanding, thinking about and designing organizational systems that are capable of responding to and influencing complex nonlinear relationships. Understanding the local dynamics in a complex system can provide great insight into the behavior of the overall system and help identify key leverage points of change and transformation."

- Trans-disciplinary approaches: "Complexity science is truly an interdisciplinary science" write Irene Sanders and Judith McCabe. "Adopting a systems view of the world meant that the questions were too big for any one discipline alone to answer. As scientists began looking for connections among the different types of complex systems, the boundaries between disciplines began to open. As a result we are witnessing the integration of knowledge across the disciplines – the physical sciences, social sciences and the humanities. Insights about complex systems are emerging across a broad spectrum of fields - from physics, mathematics and computer science, to biology, oceanology, neuroscience, art and architecture. From health care to city planning, complexity science is creating a fundamental shift in the way we view the dynamics and interactions of complex systems." This the real message of complexity science (in the sense of Marshall McLuhan's "The medium is the message.). Complexity science is the "medium" through which multiple disciplines can together transcend their parochial views, and facilitate the emergence of a new world view.

Complicated System Concepts

We need to develop strategies for people to deal with the range of complicated system they have to deal with. And, we then need to develop ways to get this knowledge to the public.

Introduction Strategies

I propose two strategies to introduce complexity science:

- Modeling: I think that teaching modeling, especially, massively parallel modeling, is an effective strategy for introducing complexity into education. First is that this type of modeling is the easiest way to understand complexity. Secondly, massively parallel modeling is easily accessible to students and educators (thanks to the work of MIT). Third, massively parallel modeling is a skill valuable to all disciplines now and in the future. And, lastly, it appears to be a good way of educating students.

  Resnick writes, "The idea of learning through design is one aspect of what Seymour Papert has called the *constructionist* approach to learning and education. Constructionism involves two types of construction. First, it asserts that learning is an active process, in which people actively construct knowledge from their experience in the world. (This idea is based on the *constructivist* theories of Jean Piaget.) To this, constructionism adds the idea that people construct new knowledge with particular

effectiveness when they are engaged in constructing products that are personally meaningful."

Models play important roles in the development of knowledge. Mitchell lists five:

- Show that a proposed mechanism for a phenomena is plausible or implausible
- Explore the effects of variations on a simple model and prime one's intuitions about a complex phenomenon
- Inspire new technologies
- Lead to mathematical models

Colella, Klopfer and Resnick, ***Adventures in Modeling***, have developed a curriculum for teaching massively parallel modeling through on screen and off screen activities and challenges using the StarLogo[3] modeling system. They write, "Research has shown that the process of creating models (as opposed to simply using models built by someone else) not only fosters model-building skills but also helps develop a greater understanding of the concepts embedded in the models. When you build your own models, you can decide what topic you want to study and how you want to study it."

Sanders and McCabe suggest that "Agent-based modeling could be used to study the U.S. Educational system." this type of modeling "could then make it possible to study the effects of various changes on the system."

- Fractals: Fractals offer an exciting new way to teach some new concepts about geometry. Moreover, it is an extremely valuable skill. Almost all objects in nature are fractals. Fractals are a way to more realistically animate natural scenes. Fractals can be

[3] StarLogo is an agent-based simulation language developed by Mitchel Resnick, Eric Klopfer, and others at MIT Media Lab and MIT Teacher Education Program. It is an extension of the Logo programming language, a dialect of Lisp. Designed for education, StarLogo can be used by students to model the behavior of decentralized systems. StarLogo TNG (The Next Generation) version 1.0 was released in July 2008. It provides a 3D world using OpenGL graphics and a block-based graphical language to increase ease of use and learning. Both are offered free and run on a personal computer.

StarLogo is a computer modeling tool that empowers students to understand the world through the design and creation of complex systems models. StarLogo enables students to program software creatures to interact with one another and their environment, and study the emergent patterns from these interactions. Building an easy-to-understand, yet powerful tool for students required a great deal of thought about the design of the programming language, environment, and its implementation. The salient features are Star Logo's great degree of transparency (the capability to see how a simulation is built), its support to let students create their own models (not just use models built by others), its efficient implementation (supporting simulations with thousands of independently executing creatures on desktop computers), and its flexible and simple user interface (which enables students to interact dynamically with their simulation during model testing and validation). The resulting platform provides a uniquely accessible tool that enables students to become full-fledged practitioners of modeling. In addition, we describe the powerful insights and deep scientific understanding that students have developed through the use of StarLogo.

StarLogo is part of the Scheller Teacher Education Program and K-12 Education Projects at MIT. StarLogo and StarLogo TNG have their own web sites, supporting information, examples and support groups. In addition, a StarLogo TNG Educational Games social network has been formed.

designed to depict things not in nature, and to quickly “draw” a realistic picture of a natural object. Moreover, they can be beautiful and can be admired as works of art. Fractals can be taught as the bridge between the mathematics of complexity and geometry, complexity and art, and art and science, among other things. The texture of a line or brush stroke gives a drawing character not accessible through today's personal computer. I suspect that the character is the line's fractal nature.

Focus Areas

There are several problems that potentially might act as focus points for the education effort:

- Ecology: Ecology is a natural choice for a focus of education. All ecological systems are by nature complex and trans-disciplinary. However, ecology has become a political issue and may have lost its impact as a good focal point. Where I still think that there is potential is the application of complexity science to smaller scale problems. There is so much misinformation in the public domain about the ecology of smaller systems like wind power is pollution free, electric cars don't pollute, or the new fluorescent bulbs (to replace incandescent bulbs) are good for the environment.
- Economics: Economics is another natural because the systems are complex and the economy is on everyone's mind. However, the problem is very complex, essentially unsolved and there is considerable controversy over the application of complexity science to economics. On the other hand, a trans-disciplinary approach would be very beneficial to help students understand their role in helping define the new economy, a future that is unfolding.
- Democracy: John Lebkowsky wrote in the introduction of ***Extreme Democracy,*** “In the 1990s online activists experimented with the Internet and the World Wide Web as a platform for a new kind of politics, leveraging interactive "many-to-many" tools to support both advocacy and deliberation. Early online activism focused on issues that were relevant to the Internet's strong "geek" element, “cyber liberties" issues of free speech and privacy. However in 2000, as Internet penetration was mainstreaming and reaching critical mass, the web became relevant to political campaigns. In the presidential campaign for election 2004, the Internet became an essential part of the political process. Howard Dean's short-lived front-runner status, a product of his campaign's effective use of Internet tools, proved that the Internet could have an effect. Though Dean was unsuccessful in his bid for the Democratic nomination, he continued to use web-based tools effectively to take control of the Democratic Party.” The involvement of technology in politics has continued and played a large role in the successful campaign of Barack Obama. However, the real potential is the application of complexity science not to politics but to improving the democratic process. This is a trans-disciplinary problem with real applications, but, it is charged with emotion, and of course politics.

There are many potential focal points for the teaching of complexity science and the ones mentioned above can be considered “low hanging fruit”. However, they demonstrate the charged environment into which any complexity science education effort will be launched. Complexity science is by its very nature transformational.

Closing

There are four things that must be an integral part of any complexity science education initiative:

1. The message and the medium must be consonant. If you're going to teach principles of complexity science, the method of teaching must be based on those same principles. That implies, among other things, that the system into which this initiative is directed must include all of the key elements affecting education, not just the students.
2. Complexity science is transformational. By many factions it will be viewed as a threat, something to be feared
3. Complexity science is strange. The principles of complexity science do not agree with what we have been taught, how we experience the world and what our intuition tells us about how things work. Concepts like cause not linked to effect, non-predictability, extreme sensitivity to initial conditions, non-linearity, non-reversibility, emergence of complexity from simple rules, and paradox (among others). “Complexity science is highly paradoxical. As you study the world through a complexity lens you will be continually confronted with 'both-and' rather than 'either-or' thinking. The paradoxes of complexity are that both sides of many apparent contradictions are true.” comments Zimmerman, Lindberg and Plesk.
4. It will be a slow process. Ralph Abraham describes one of limitations of the rate of acceptance of a new idea like complexity science, “According to 'folk theory' circulating among mathematicians, a mathematical object will emerge into the collective consciousness of a group somewhat after the complexity of the group mind evolves to the necessary minimum level to support the cognition of that object. That is, the brain and mind evolve, consciousness evolves, mathematical skills evolve, language evolves, all of these coevolve. According to this theory mathematical objects are 'discovered' as soon as people are able to understand them.”

I'll close this essay by quoting Warren Weaver in the same article he wrote when he introduced the concept of organized and disorganized complexity in 1948. I don't believe that complexity science will solve all the problems he lists, but I am positive that complexity science is going to open up new ways of thinking about the intractable problems of modern life.

“The great gap, which lies so forebodingly between our power and our capacity to use power wisely, can only be bridged by a vast combination of efforts. Knowledge and individual group behavior must be improved. Communication must be improved between peoples of different languages and cultures, as well as all the varied interests which use the same language, but often with such dangerously different connotations. A revolutionary advance must be made in our understanding of economic and political factors. Willingness to sacrifice selfish short-term interests, either personal or national, in order to bring about long-term improvement for all must be developed.”

I am excited and optimistic about the future of complexity science. It is unfinished work but the challenge and potential is great. Rather than looking at the problems of complexity science I fall more into the camp of one of characters in ***Arcadia***, a 1993 play by Tom Stoppard concerning the relationship between past and present and between order and disorder and the certainty of

knowledge.

"It makes me happy. To be at the beginning again, knowing almost nothing...The ordinary sized stuff which is our lives, the things that people write poetry about – clouds – daffodils – waterfalls...these things are full of mystery, as mysterious to us as the heavens were to the Greeks...It's the best possible time to be alive, when almost everything you thought you knew is wrong."

Resources

1. Complexity and Education, This website is intended as a meeting place for educators and educational researchers whose work is informed and/or orientated by complexity, http://www.complexityandeducation.ualberta.ca/
2. StarLogo on the Web, http://education.mit.edu/starlogo/
3. StarLogo TNG Educational Games, http://starlogo-tng-educational-games.ning.com/
4. John Conway's "The Game of Life", http://www.bitstorm.org/gameoflife/
5. "Resources for Understanding and Applying Complexity: Book Summaries", Paul Schumann, September 2009, http://incollaboration.ning.com/profiles/blogs/resources-for-understanding
6. "Paper Weight: The health care bill is more than 1,000 pages. Is that a lot?", Christopher Beam, August 20, 2009, http://www.slate.com/id/2225820/?from=rss
7. "Complexipacity: A Neologism for Modern Times", David Snyder, July 2009, http://www.wfs.org/wf09Complexipacity.pdf
8. "Chaos: A Brief Introduction", Greg Rae, IMHO In My Humble Opinion, undated blog, http://imho.com/grae/chaos/chaos.html
9. ***Complexity: A Guided Tour***, Melanie Mitchell, Oxford, 2009
10. ***Simplexity***, Jeffrey Kluger, Hyperion, 2008
11. ***Edgeware: Insights from Complexity Science for Health Care Leaders***, Brenda Zimmerman, Curt Lindberg and Paul Plesk, Lindberg Publishing, 2008. "A Complexity Science Primer: What is Complexity Science and Why Should I Learn About It?" Available at http://www.napcrg.org/Beginner%20Complexity%20Science%20Module.pdf
12. ***The Black Swan: The Impact of the Highly Improbable***, Nassim Nicholas Taleb, Random House, 2007
13. Complex Systems: Network Thinking, Melanie Mitchell, http://web.cecs.pdx.edu/~mm/AIJ2006.pdf
14. ***Extreme Democracy***, edited by Mitch Ratcliffe & Jon Lebkowsky, 2005, http://extremedemocracy.com/ and http://incollaboration.ning.com/profiles/blogs/extreme-democracy-discussion

15. "The Use of Complexity Science", T. Irene Sanders and Judith McCabe, Washington Center for Complexity & Public Policy, 2003 Can be found on the Internet at http://www.complexsys.org/pdf/ComplexityScienceSurvey.pdf

16. "StarLogo: Under the Hood and in the Classroom", Eric Klopfer and Andrew Begel, Kybernetes, 2003, Volume:32, Issue:1/2

17. ***Adventures in Modeling: Exploring Complex, Dynamic Systems with StarLogo***, Vanessa Stevens Colella, Eric Klopfer & Mitchel Resnick, Teachers College press, 2001

18. ***Emergence: The Connected Lives of Ants, Brains, and Software***, Steven Johnson, Scribner,2001

19. ***Ubiquity: Why Catastrophes Happen***, Mark Buchanan, Three Rivers Press, 2000

20. ***Turtles, Termites and Traffic Jams: Explorations in Massively Parallel Microworlds***, Mitchel Resnick, MIT, 1997

21. ***Chaos Gaia Ero***s, Ralph Abraham, Harper, 1994

22. ***Tao of Chaos: Merging East and West***, Katya Walter, Kairos Center, 1994

23. ***Arcadia***, Tom Stoppard, 1993; print version Faber & Faber; First Edition, 1994; description on Wikipedia, http://en.wikipedia.org/wiki/Arcadia_%28play%29

24. ***Complexity: The Emerging Science at the Edge of Order and Chaos***, M. Mitchell Waldrop, Touchstone, 1992

25. ***Exploring Complexity: An Introduction***, Gregoire Nicolis & Ilya Prigogine, Freeman, 1989

26. ***Chaos: Making a New Science***, James Gleick, Penguin, 1987

27. **Gödel, Escher, Bach: An Eternal Golden Braid**, Douglas Hofstadter, Vintage, 1980

28. "Managing Modern Complexity", Stafford Beer, presentation to the Committee on Science and Astronautics of the House of Representatives, Washington, DC, January 27, 1970, in ***Platform for Change***, John Wiley, 1975

29. "Science and Complexity", Warren Weaver, American Scientist, 36: 536 (1948) Also available on the Internet at http://www.ceptualinstitute.com/genre/weaver/weaver-1947b.htm